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OpenSense Short Online Conference (SoC)

How can Signal Processing Enhance Opportunistic Precipitation Sensing?

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TEL AVIV UNIVERSITY
Pursuing the Unknown

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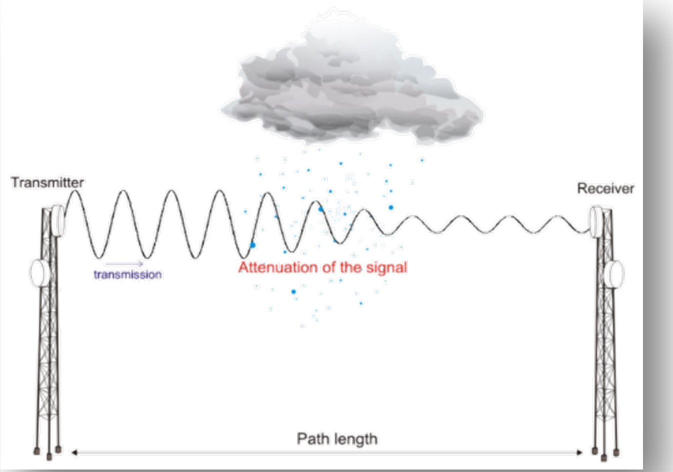
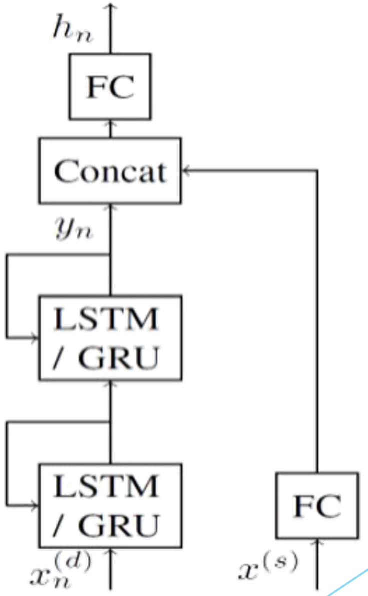
Local Rainfall **estimation**: transforming measurements from a single **CML** into rain information (VRG)



Data Driven:

- Labeled data used are rain gauges close to CMLs (up to 2 Km)
- Training is climate sensitive



$$\hat{R} = \left(\frac{\int_0^L R(x)^b dx}{L} \right)^{1/b}$$

$$\bar{R} = \frac{1}{L} \int_0^L R(x) dx$$

Model Driven:

- Calibration of the model's parameters;
- Wet/dry classification;
- Extraction of the rain-induced attenuation;
- Applying a simplified model:

$$A[dB] = aR^b L$$

Habi, Hai Victor, and Hagit Messer. "Wet-Dry Classification Using LSTM and Commercial Microwave Links." *2018 IEEE 10th Sensor Array and Multichannel Signal Processing Workshop (SAM)*. IEEE, 2018.

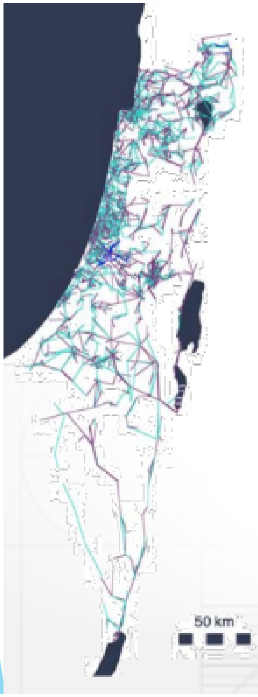
How can Signal Processing Help us?

In today's talk, we will present and answer three questions:

- How much performance is lost in rain-field reconstruction when treating CMLs as a single virtual rain gauge (located in their mid-point)?
- How much rain-rate information can be extracted from min-max signal-level samples?
- How is the CMLs network topology related to the rain-field reconstruction resolution?

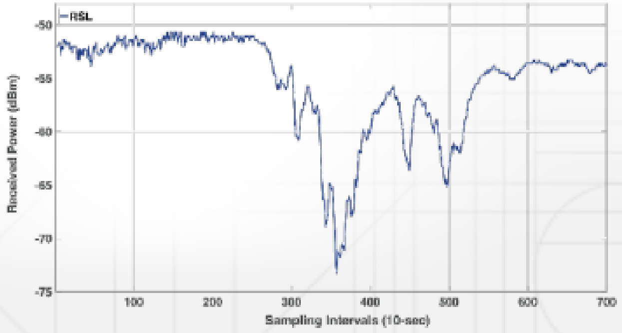
1. How much performance is lost in rain-field reconstruction when treating CMLs as a single virtual rain gauge (located in their mid-point)?

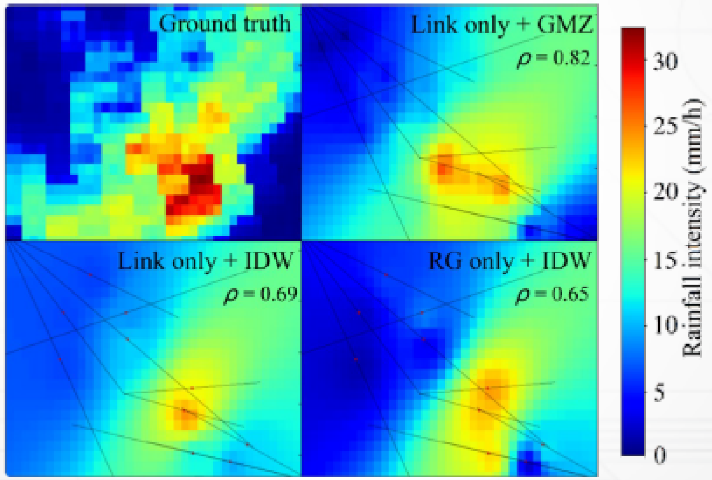
2-D rain-mapping: transforming a wireless network to a virtual weather radar



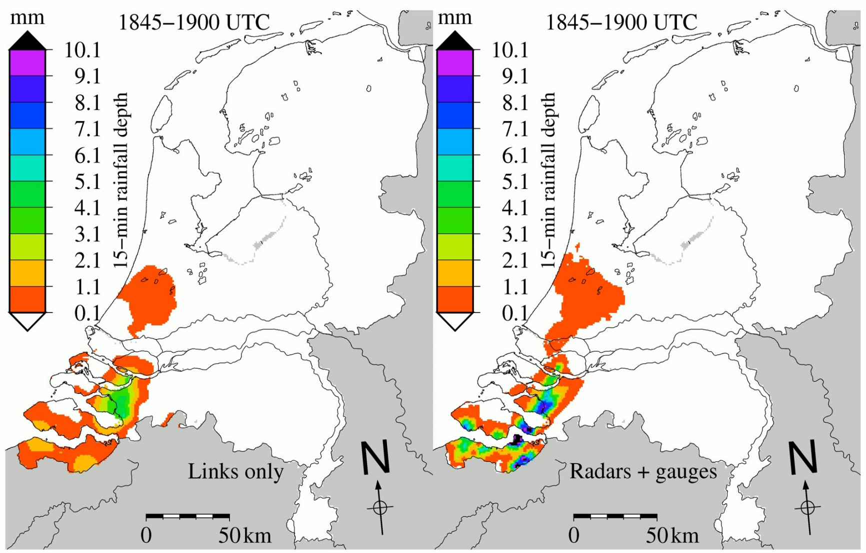
Commercial microwave links are attenuated by rain

The received power levels of the attenuated channels hold information about the rain that can be extracted





Advanced signal processing and machine learning Algorithms are used to produce accurate rainfall maps



Overeem, Aart, Hidde Leijnse, and Remko Uijlenhoet. "Country-wide rainfall maps from cellular communication networks." *Proceedings of the National Academy of Sciences* 110.8 (2013): 2741-2745.

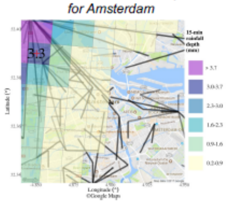
Rainfall observation using commercial microwave links (CMLs): An overview of ongoing projects around the globe

Christian Chwala, Hidde Leijnse, Aart Overeem, Remko Uijlenhoet, Hagit Messer, Pinhas Alpert, Ali Doumounia, François Zougmoré, Martin Fencl, Vojtěch Bareš, Muhammad Sohail Afzal, Syed Hamid Hussain Shah, Giacomo Roversi, Pier Paolo Alberoni, Remco van de Beek, Jafet Andersson, Carlo De Michele, Michele D'Amico, Roberto Nebuloni, Marielle Gosset, Harald Kunstmann

Netherlands (overeem@knmi.nl)

- Years of CML data from T-Mobile NL
- via ftp (1 h – 1 d delay)
- Up to thousands of CMLs
- Min/max RSL 15 min or inst. RSL 15 min

CML-based rainfall map for Amsterdam

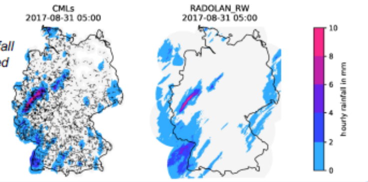


- Open source code CML rainfall retrieval
  https://github.com/overeem11/RAINLINK
- 2.5 years of country-wide rainfall maps
  https://doi.org/10.1002/2016WR019412
- Overview of CML rainfall estimation
  https://doi.org/10.1002/wat2.1289

Germany (christian.chwala@kit.edu)

- 4000 CMLs with 1-min instantaneous data
- Real-time DAQ via open source software
  https://doi.org/10.5194/amt-9-991-2016
- Open source Python processing toolbox
  https://github.com/pycomlink/pycomlink
- Focus on countrywide real-time rainfall and application in hydrology

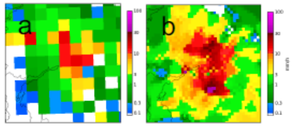
Hourly CML rainfall vs gauge adjusted radar product RADOLAN 1km x 1km



Sweden (jafet.andersson@smhi.se) remco.vandebeek@smhi.se)

- 10sec data from 364 CMLs
- Operational prototype of 1-min rainfall over Gothenburg since 2016 (updated hourly)
  https://www.smhi.se/en/services/professional-services/micro-weather-live-data/
- Microwave Links Improve Operational Rainfall Monitoring in Gothenburg, Sweden
  https://cest2017.gnest.org/sites/default/files/presentation_file_list/cest2017_00249_oral_paper.pdf
- Partners:

Hi3G
ERICSSON

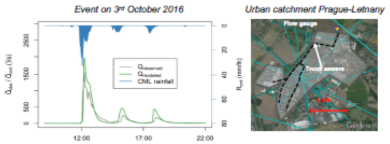


Gothenburg city during a rainfall event (2015-07-08 00:45 UTC) based on the radar (a, 4km2 × 15-min. resolution and CMLs (b, 0.25km2 × 1-min resolution.

Czech Republic (vojtech.bares@cvut.cz)

- 400 CMLs (from three cities)
- T-Mobile, Czech Republic
- Collected at $\Delta t \approx 10$ s
- Focus on urban hydrology
- Collaboration with water utilities

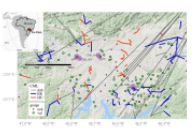
Observed and simulated in-sewer runoff from 19 CMLs:



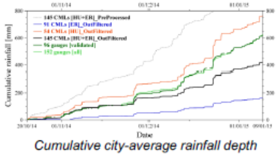
Open source code for CML processing: https://github.com/fenclmar/Rcmlrain
Runoff modelling with CML data: https://doi.org/10.2166/wst.2018.149
Adjusting CMLs with rain gauges: https://doi.org/10.5194/hess-21-617-2017

Brazil (overeem@knmi.nl)

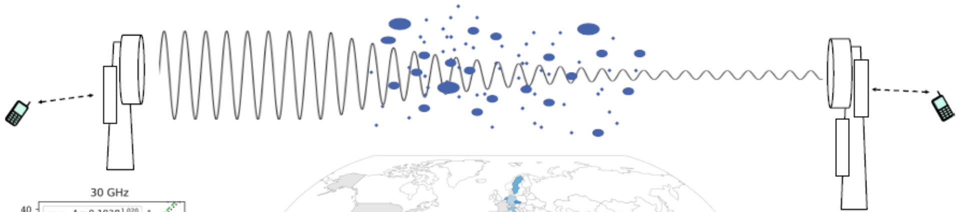
- 3 months of CML data from TIM for São Paulo
- A few hundred CMLs
- Min/max or only min RSL every 15 min
- Processing challenges due to mixed quality of metadata
  https://doi.org/10.5194/amt-11-4465-2018

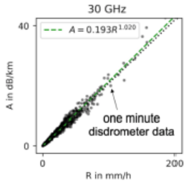


Map of CMLs in for São Paulo



Cumulative city-average rainfall depth



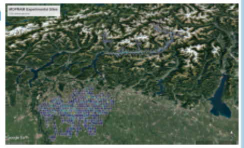


Robust relation between observed attenuation an rain rate

Italy (carlo.demichele@polimi.it)

- Partners: Politecnico di Milano and CNR (Consiglio Nazionale delle Ricerche)
- Cooperation: SIAE microelettronica, VODAFONE ITALIA
- CML number: 20 (Sondrio) + 100 (Milan)
- Time resolution: 15 min

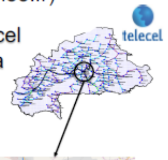
Number of available CMLs per pixel in the two experimental areas in Lombardy: Milan metropolitan area (bottom left) and Sondrio Alpine area (upper center)

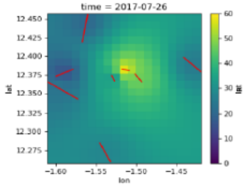


Tomographic reconstruction of CML rainfall maps: https://doi.org/10.1109/LGRS.2016.2614326
More info on MOPRAM and publications: http://www.mopram.it/

Burkina Faso (doumouniaali@yahoo.fr)

- Number of CMLs: 586 for Telecel
- Resolution: 1min real time data
- Partners : ANAM, Telecel, Telmob, WASCAL, UO1, IDS



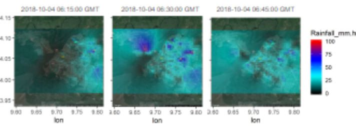


Cameroun and Niger (marielle.gosset@ird.fr)

- Data in Niger/Niamey since 2016
- Real-time rainfall maps in Cameroun since 2018

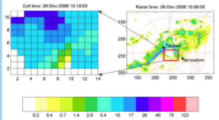
- Flux of real-time raw data at 15 minute step (min/max/mean)
- Based on Orange SAM Network Monitoring System
- Hydrological applications

Dense CML network over Douala



Israel (messer@eng.tau.ac.il, pinhas@post.tau.ac.il)

- Country wide measurements from all cellular providers (since 2005)
- Typical NMS measurements (daily or 15-min)
- Various algorithms for rain mapping
- CML-derived air moisture and mapping

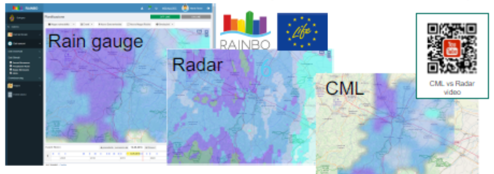


Rainfall maps from CML (left) vs. Radar (right)

Preliminary results for CML-based air moisture mapping

Italy (giacomo.roversi2@studio.unibo.it)

- Area: Bologna, Parma, Emilia Romagna region
- Historical data set: 751 CMLs (15', min/max)
- Real-time data: 346 CMLs (1', inst)
- RainBO off-line mode: Maps of rainfall depth
- RainBO on-line mode: Rain rate per CML
- Software: Rainlink4EMR (based on RAINLINK)



CML vs Radar video

IPWG 2018, Seoul, 05.11.2018

How CML-based rain maps are created?

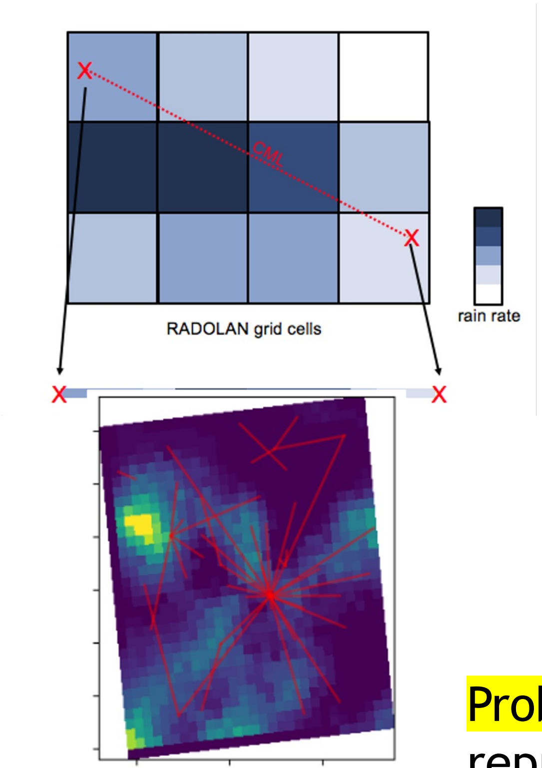
- The most commonly used methods consist of:

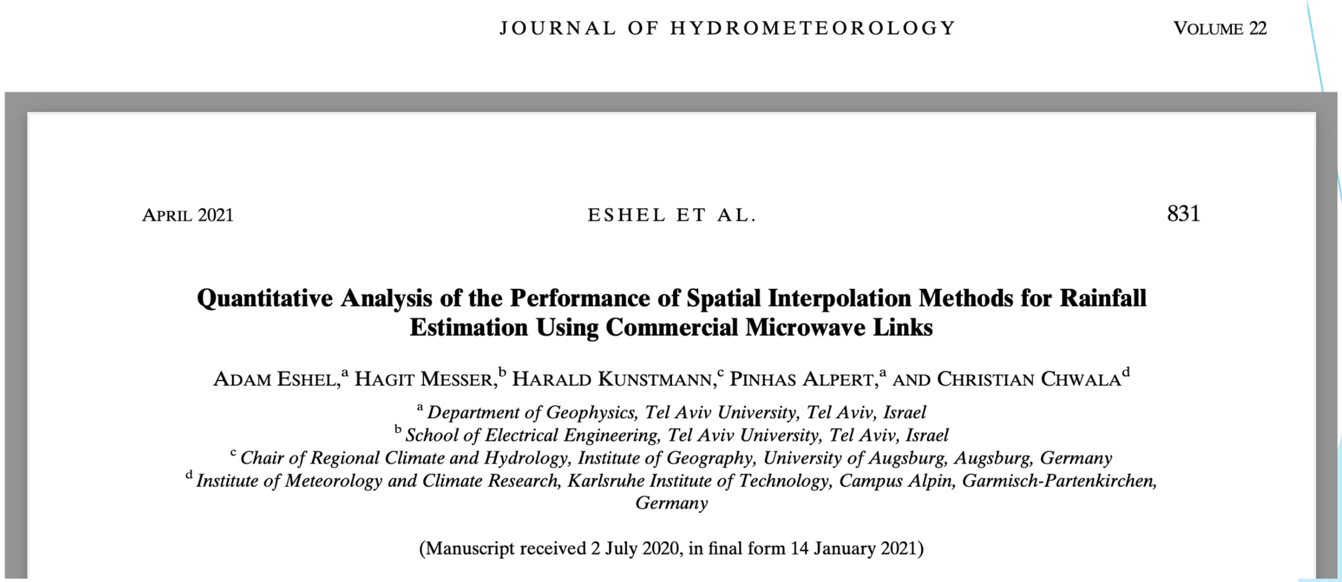
1. **Preprocessing** of the CML measurements (isolating the rain-induced component in the signal for each CML and then converting attenuation to rain using the power-law relation: $A[dB] = aR^b L$
2. Assuming that $\hat{R} = \sqrt[b]{\frac{A}{aL}}$ is **a point measurement** in the middle of the link, a 2-D rain map is constructed using measurements from multiple CMLs in the area using standard **spatial interpolation** techniques, e.g. Kriging or IDW.
3. *This approach neglects the spatial spread of a CML (length, orientation).*
4. The question: how much does one lose by ignoring this information?

How to approach this question?

- **Environmental sciences approach:** empirical comparison with real data

Semi-real data study: Retrieving CML attenuation from radar to study the accuracy of naïve reconstruction methods vs. GMZ:





JOURNAL OF HYDROMETEOROLOGY VOLUME 22

APRIL 2021 ESHEL ET AL. 831

Quantitative Analysis of the Performance of Spatial Interpolation Methods for Rainfall Estimation Using Commercial Microwave Links

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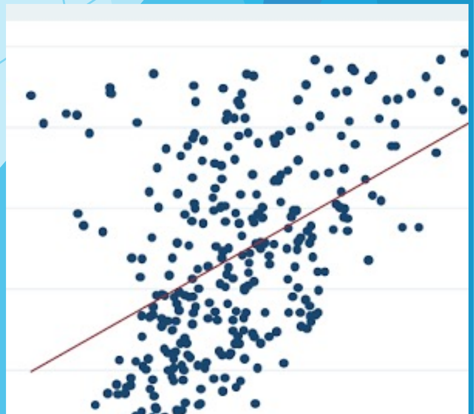
[b] *School of Electrical Engineering, Tel Aviv University, Tel Aviv, Israel*
[c] *Chair of Regional Climate and Hydrology, Institute of Geography, University of Augsburg, Augsburg, Germany*
[d] *Institute of Meteorology and Climate Research, Karlsruhe Institute of Technology, Campus Alpin, Garmisch-Partenkirchen, Germany*

(Manuscript received 2 July 2020, in final form 14 January 2021)

Problems: results are **algorithm/scenario dependent** often represented by a scatter plot against processed data (no ground truth) so conclusions are subjective



How to approach this question?

- **Signal processing** approach: theoretical analysis based on statistical model

Parameter Estimation Approach:
Given measurements vector X with a PDF $p(X|\theta)$ where θ is a parameter vector to be estimated such that $\hat{\theta} = \text{g(X)}$ and g is the estimation algorithm, then the estimation error is the covariance of $\epsilon = \theta - \hat{\theta}$.

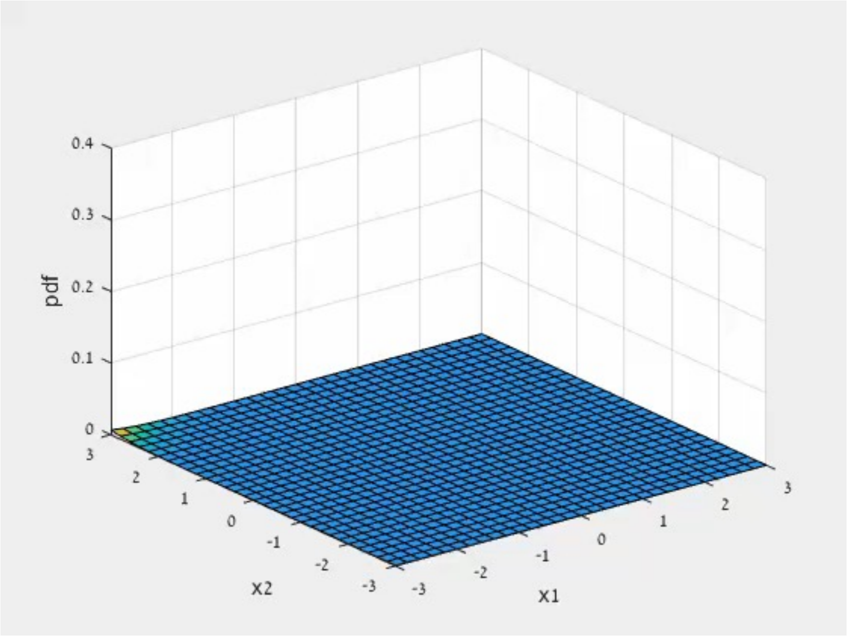
The Cramer-Rao Bound (CRB) inequality states that for any g,
$COV(\epsilon) - FIM^{-1} \geq 0$ where FIM is the Fisher Information Matrix given by:

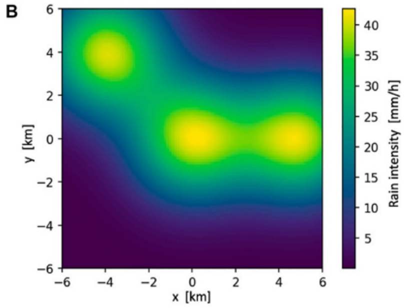
$$FIM_{ij} = E\left\{\frac{\partial \ln p(\frac{x}{\theta})}{\partial \theta_i}\frac{\partial \ln p(\frac{x}{\theta})}{\partial \theta_j}\right\}$$

Thus, the CRB= FIM^{-1} indicates on the best possible (optimal) performance, independent on the algorithm used.

Parametric rain-field modeling

▸ Gaussian Shape

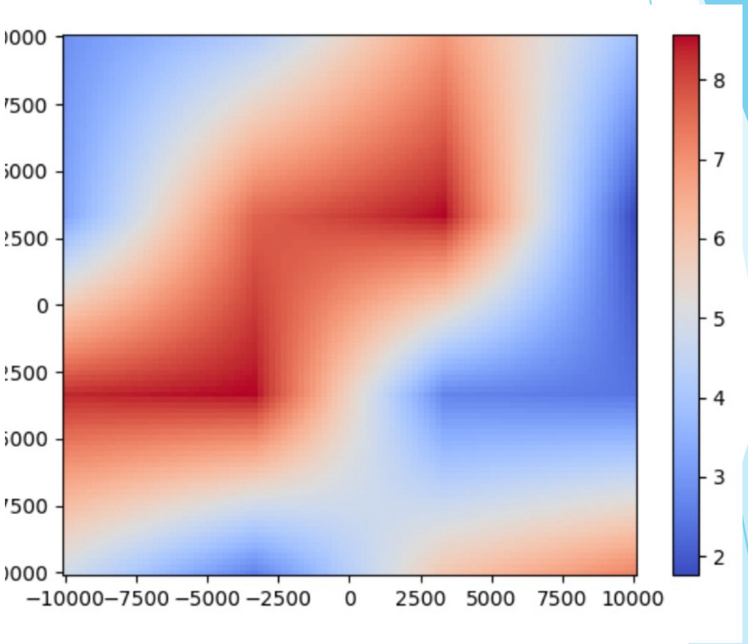


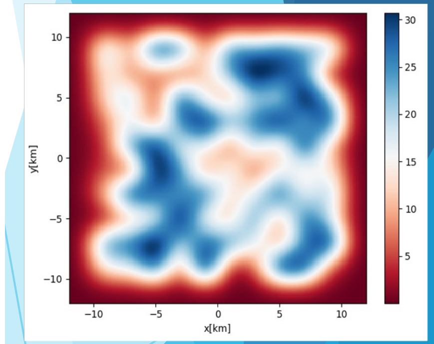


$$\boldsymbol{\theta} = [R_0, \mu_{0x}, \mu_{0y}, \sigma_x^2, \sigma_y^2, \rho, v_x, v_y]$$

$$R(x,y,t;\boldsymbol{\theta}) = R_0 e^{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-(\mu_{0x}+v_x t))^2}{\sigma_x^2}+\frac{\left(y-(\mu_{0y}+v_y\cdot t)\right)^2}{\sigma_y^2}-\frac{2\rho\left(x-(\mu_{0x}+v_x t)\right)\cdot\left(y-(\mu_{0y}+v_y t)\right)}{\sigma_x\sigma_y}\right]}$$

▸ B-Spline





$$\boldsymbol{\theta} = [P_{00}, P_{01}, \ldots, P_{mn}, v_x, v_y]$$

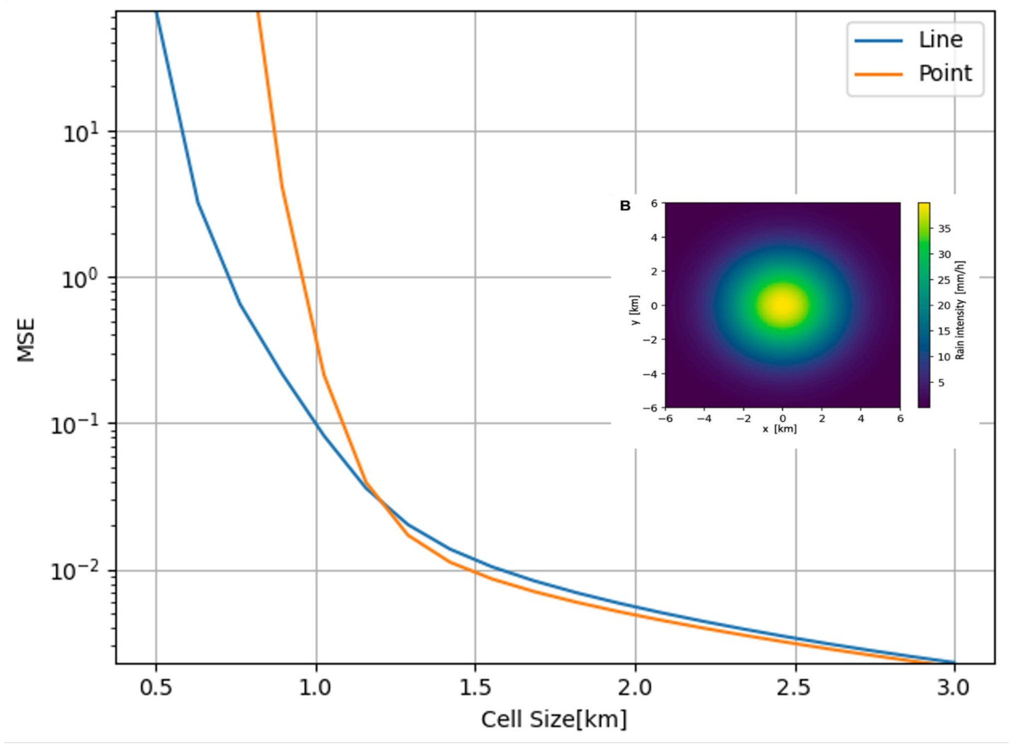
$$r(x,y,t) = \sum_{i=1}^{m}\sum_{j=1}^{n} P_{i,j} B_i^k(x - v_x t) B_j^l(y - v_y t)$$

CRB study of the difference between point (RG) and line (CML) ground level sensors in random topology for rain map estimation

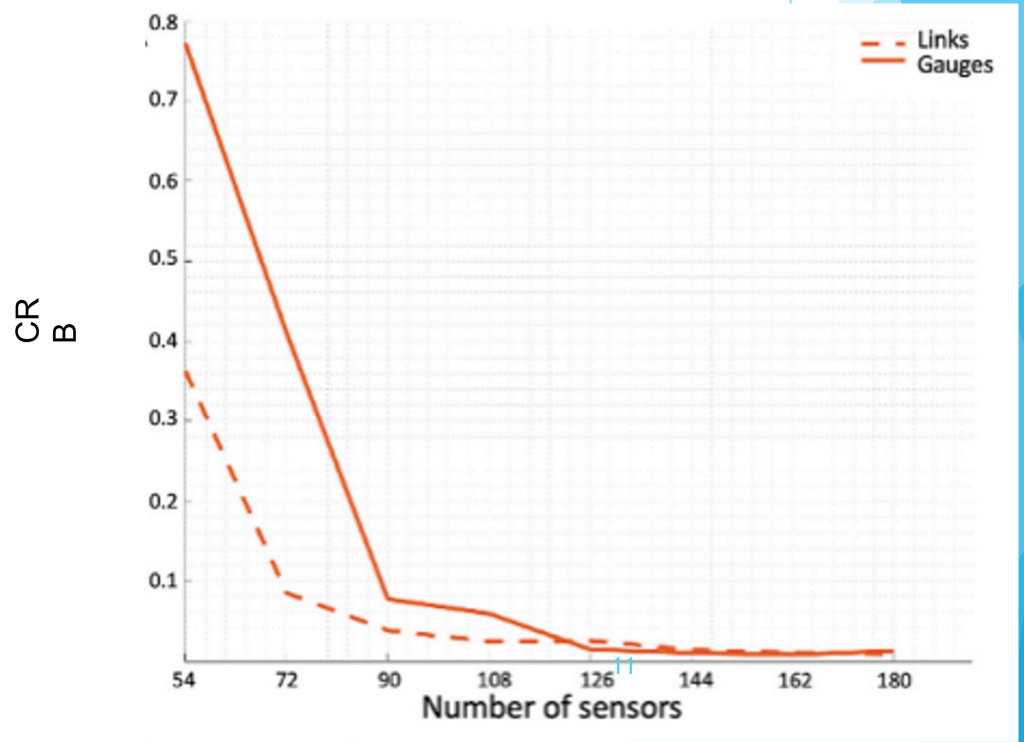
A simplified measurements model:

- Line Sensor (Link) $X = \int_{s=0}^{L} a \cdot r(x_L(s), y_L(s); \theta)^b ds + W$
- Point Sensor (Gauge) $Y = r(x_p, y_p; \theta) + N$

Gaussian-Shape



B-Spline



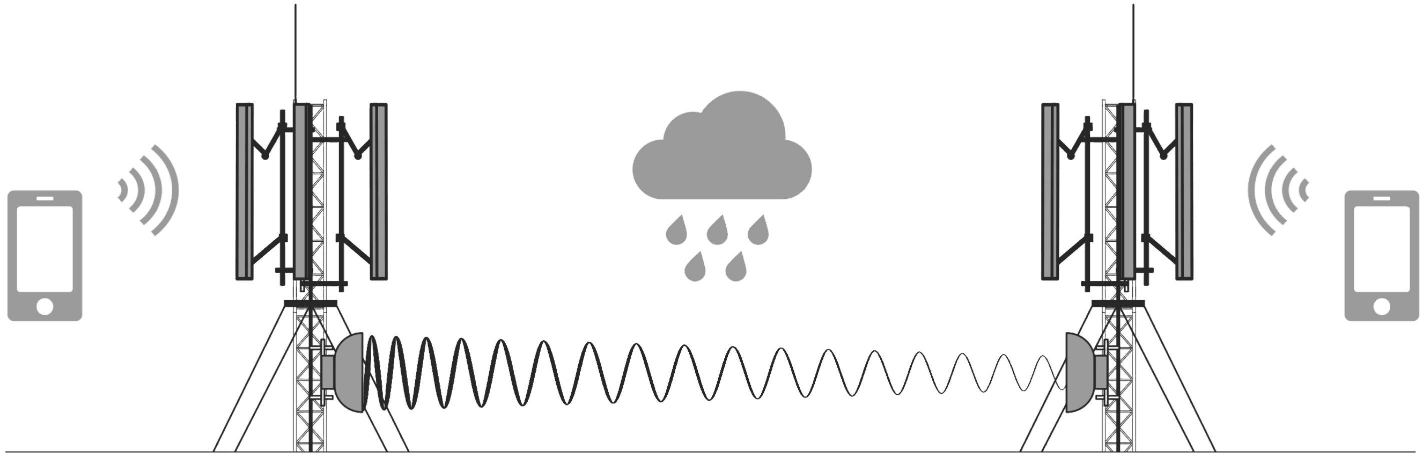
PERFORMANCE ANALYSIS (ACCURACY) – THEORETICAL, BY SIMULATIONS AND EXPERIMENTAL STUDIES

- Sagiv S., Messer H. (2025): **Lower Bound Analysis of Parameter Estimation of Moving Field by Sensors in Random Locations**, IEEE SSP'25, theoretical
- Habi HV., Messer H. (2025): **Optimal Allocation of Auxiliary Designated Sensors for Opportunistic Rain Field Reconstruction,** IEEE SSP'25 theoretical
- MESSER H., Eshel A., Habi H.V., Sagiv S. and Zheng X (2022). **Rain field retrieval by ground-level sensors of various types**. *Frontiers in Signal Processing* [Link]. review
- Zheng X, Messer H, Wang Q, Xu T, Qin Y, Yang T (2022). **On the potential of commercial microwave link networks for high spatial resolution rainfall monitoring in urban areas.** *Atmospheric Research, 106289* [Link]. simulations
- Eshel, A., Messer, H., Kunstmann, H., Alpert, P., Chwala, C. (2021). **Quantitative Analysis of the Performance of Spatial Interpolation Methods for Rainfall Estimation Using Commercial Microwave Links**. *Journal of Hydrometeorology*, 22(4), 831-834. [Link] experimental
- Eshel, A., Ostrometzky, J., Gat, S., Alpert, P., & Messer, H. (2020). **Spatial Reconstruction of Rain Fields From Wireless Telecommunication Networks—Scenario-Dependent Analysis of IDW-Based Algorithms.** *IEEE Geoscience and Remote Sensing Letters, 17*(5), 770-774. [Link] simulations
- Gat, S. and Messer, H., 2019, December. **A comparative study of the performance of parameter estimation of a 2-d field using line-and point-projection sensors.** In *2019 IEEE 8th International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP)* (pp. 146-150). IEEE. theoretical

2. How much rain-rate information can be extracted from min-max signal-level samples?

How does the CML's sampling protocol affect the rain estimation?

The (virtual) sensor



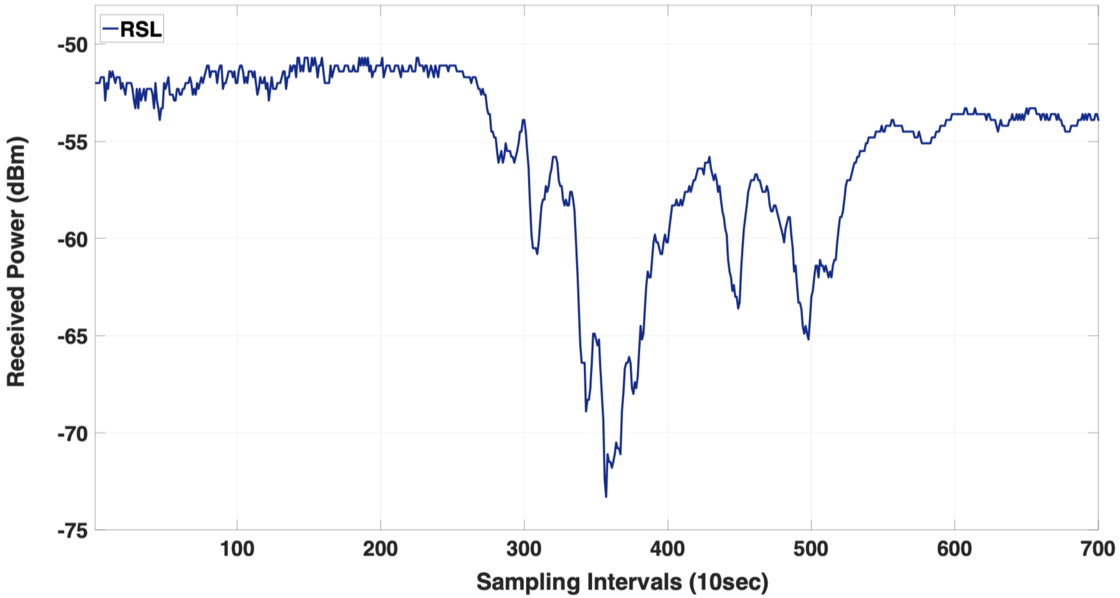
The Power-Law: $A_r(t) = a \cdot r(t)^b \cdot L \qquad (dB)$

$$r(t) = \left(\frac{A_r(t)}{aL}\right)^{1/b} \qquad \left(\frac{mm}{h}\right)$$

Instantaneous Samples

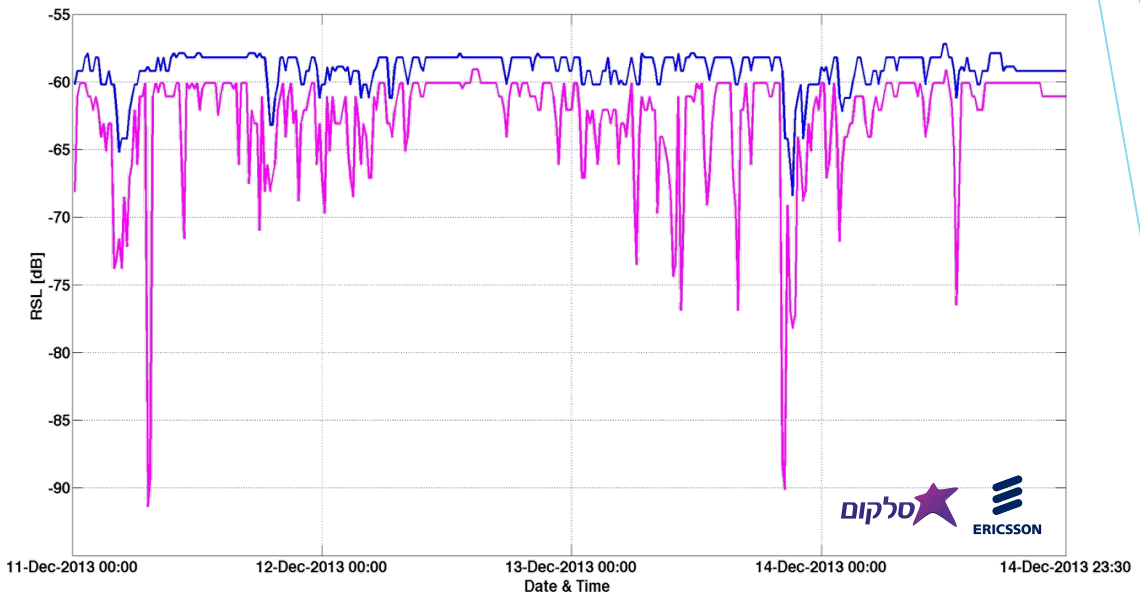
- ✓ Can be in a number of sampling-rate (e.g., every 10 seconds, every minute, every 5 minutes, every 15 minutes, etc.)
- ✓ Each sample is independent with the other samples (apart from the phenomena)

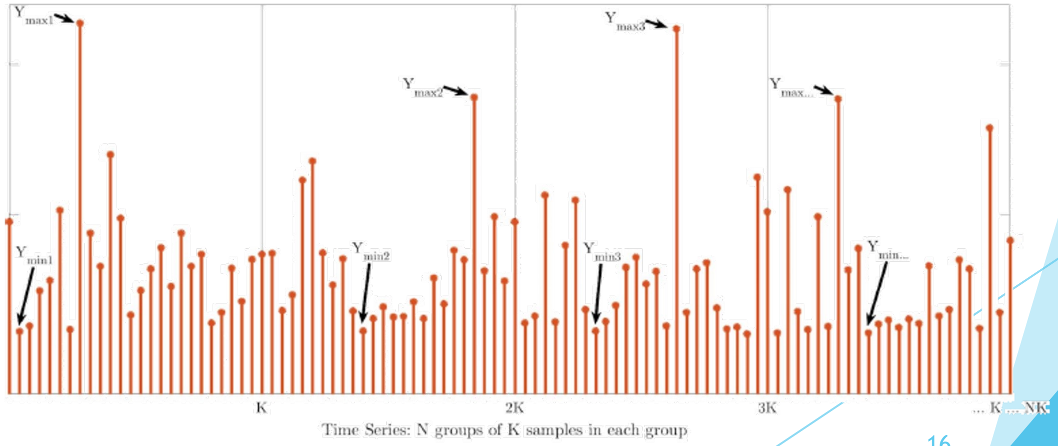
Two hours of received signal level (RSL) data-series (from an Ericsson CML in Gothenburg) during a rain event in summer 2015. The transmitted signal level (TSL) remained constant.



Min-Max Samples

- ✓ Usually based on 10-second instantaneous samples, where every 900 samples (15-minutes), the minimum and the maximum detected values are reported.
- ✓ Each min-max sample is *dependent* of all samples within the interval.
- ✓ Can be reported in other intervals.





The Challenge

- How can the min-max samples compare (rain-information wise) to instantaneous samples?

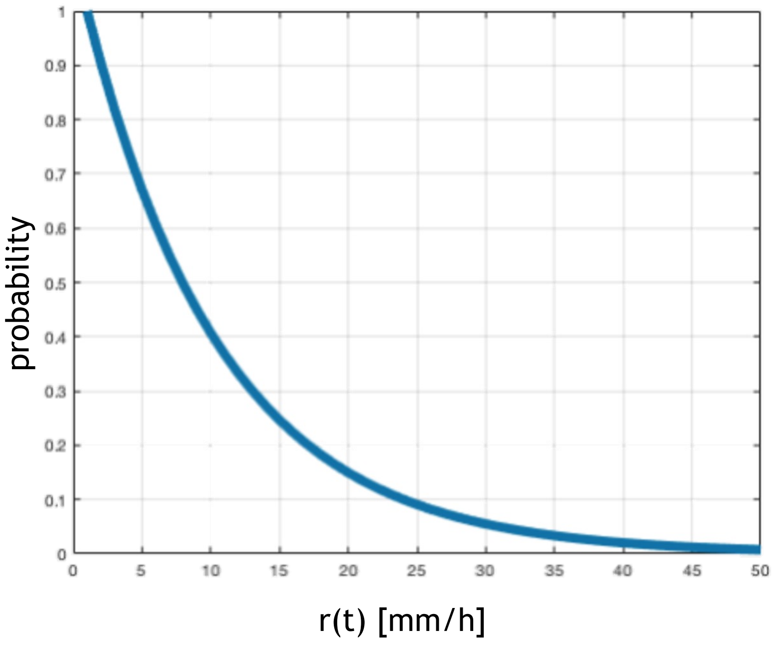
The Rain Statistics

$\Pr\{r(t) \leq r\} = F_R(r; t, \underline{\theta})$, where $f_R(r; t, \underline{\theta})$ is:

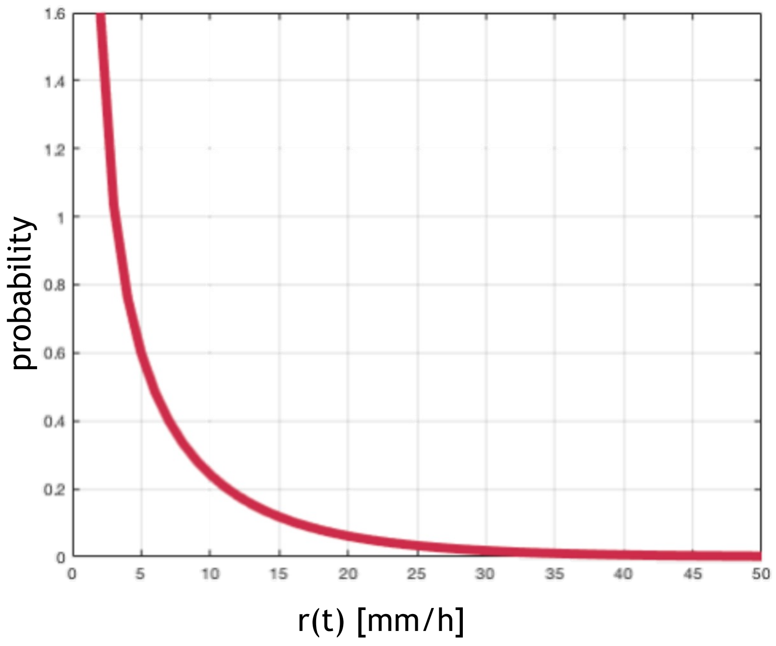
- Exponential?

Gamma?

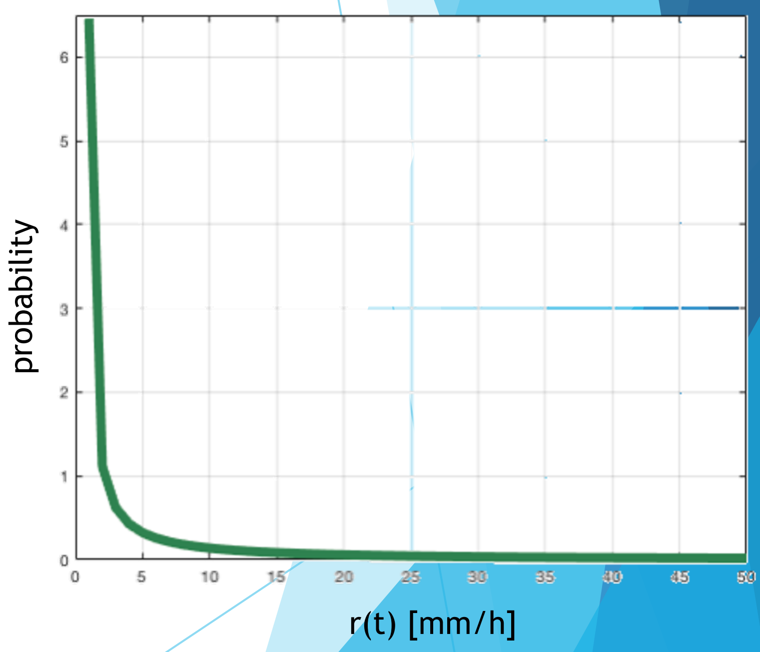
Log-Normal?



Exp(1)



Gamma(0.5,1)



LogNorm(-1,3)

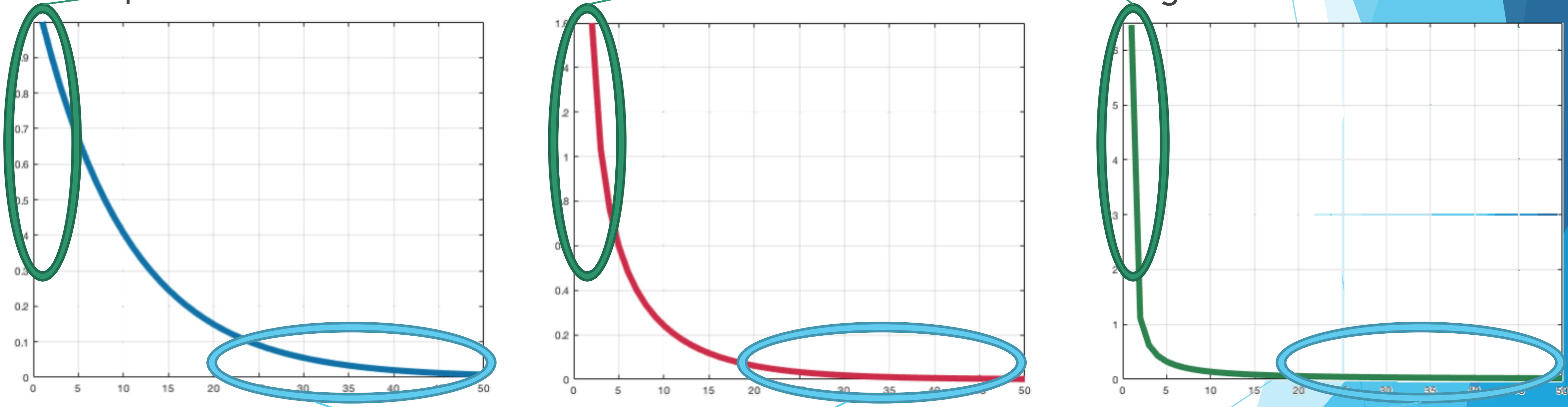
The rain statistics

The minimum is bounded (> 0)

- Exponential?

Gamma?

Log-Normal?



The maximum follows an Exponential tail

Statistics of Extreme Values

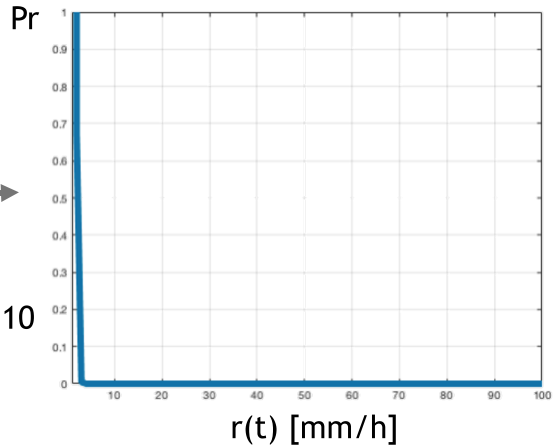
Minimum from K observations*

$$f_{Y_{min}}(z;\underline{\theta},K) = K\left[1 - F_X(z;\underline{\theta})\right]^{K-1} f_X(z;\underline{\theta})$$

$K \to \infty$

Effectively, when larger than 10

Degenerate case



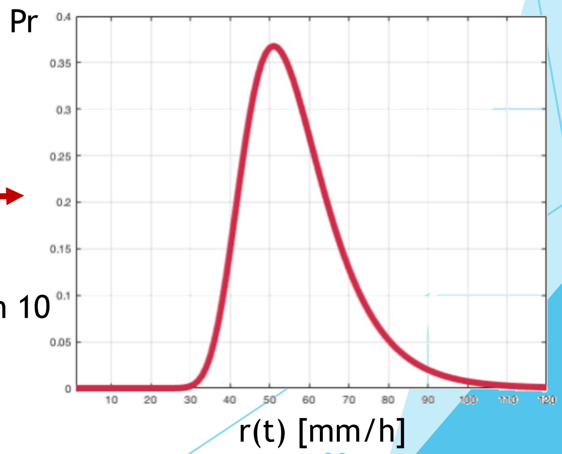
Maximum from k observations*

$$f_{Y_{max}}(z;\underline{\theta},K) = K\left[F_X(z;\underline{\theta})\right]^{K-1} f_X(z;\underline{\theta})$$

$K \to \infty$

Effectively, when larger than 10

GEV PDF Type I (Gumbel)



* Assuming iid

Ostrometzky, J., & Messer, H. (2020). On the information in extreme measurements for parameter estimation. *Journal of the Franklin Institute*, *357*(1), 748-771.

Estimation of θ

From a sequence of N minimum values

- N groups of K observations each. From each group, only the **minimum value** is taken, and used to estimate $\widehat{\theta}$:
- The Fisher Information:

$$J^{min}(\theta) = \frac{N}{\theta^2} \cdot \mathcal{O}(1)$$

From a sequence of N maximum values

- N groups of K observations each. From each group, only the **maximum value** is taken, and used to estimate $\widehat{\theta}$:
- The Fisher Information:

$$J^{max}(\theta) \cong \frac{N}{\theta^2} \cdot \mathcal{O}(ln^2(K))$$

As K increases, the information stored in the maximum values increases!
For large K, most information is stored in the maximum sequence!

$$J^{mix}(\theta) \xrightarrow[K\to\infty]{} J^{min}(\theta) + J^{max}(\theta) \cong J^{max}(\theta)$$

“mix” represents the estimation using both min and max values

What it Means is:

K	$\left[\frac{J^{\text{opt}}(\theta)}{N/\theta^2}\right]$	$\left[\frac{J^{\text{min}}(\theta)}{N/\theta^2}\right]$	$\left[\frac{\tilde{J}^{\text{max}}(\theta)}{N/\theta^2}\right]$	$\left[\frac{\tilde{J}^{\text{mix}}(\theta)}{N/\theta^2}\right]$
5	5	1	2.238	2.794
10	10	1	4.891	5.539
25	25	1	9.793	10.580
50	50	1	14.616	15.482
100	100	1	20.422	21.341
1000	1000	1	46.765	47.752

- K: The number of observations per group
- J^{opt}: The number of observations per group used in the optimal estimation process
- J^{min}: **Minimum** based estimation: the number of the **equivalent** observations per group which would result in same estimation performance (fisher wise)
- $\tilde{J}^{\text{max}}$: **Maximum** based estimation: the number of the **equivalent** observations per group which would result in same estimation performance (fisher wise)
- $\tilde{J}^{\text{mix}}$: **Mixed** based estimation: the number of the **equivalent** observations per group which would result in same estimation performance (fisher wise)

What it Means is:

K	$\left[\frac{J^{opt}(\theta)}{N/\theta^2}\right]$	$\left[\frac{J^{min}(\theta)}{N/\theta^2}\right]$	$\left[\frac{\tilde{J}^{max}(\theta)}{N/\theta^2}\right]$	$\left[\frac{\tilde{J}^{mix}(\theta)}{N/\theta^2}\right]$
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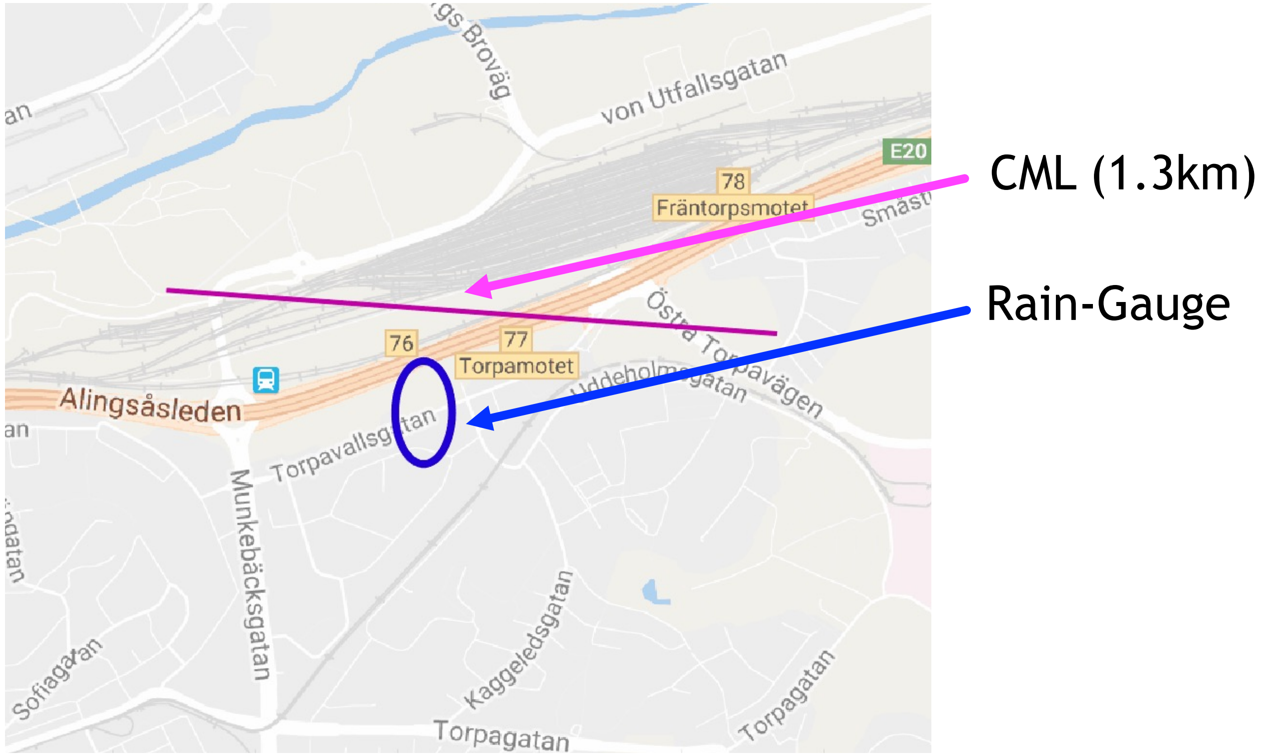
To conclude so far

- The sequence of the maximum attenuation values holds more information regarding the rain than the minimum attenuation values sequence.
- Combined, the minimum and the maximum attenuation values (reported at 15-minute intervals from 10-second sampling rate) is equivalent **up-to** 20 instantaneous samples taken within these 15 minutes!

Ostrometzky, J. (2017). *Statistical signal processing of extreme attenuation measurements taken by commercial microwave links for rain monitoring*. Tel Aviv, Israel: Tel Aviv University.

Demonstration – Specifics

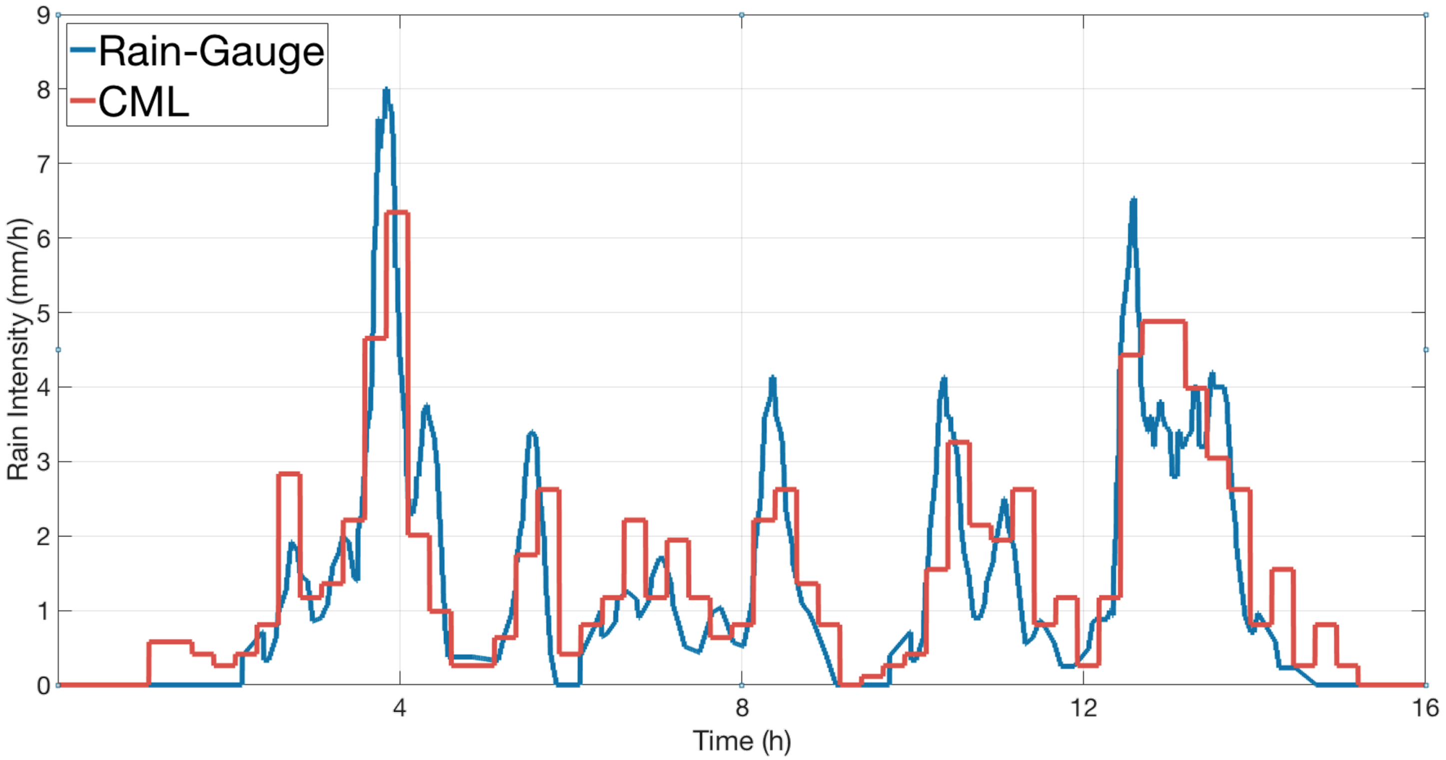


Andersson, J. C., Olsson, J., Van de Beek, R., & Hansryd, J. (2022). OpenMRG: Open data from Microwave links, Radar, and Gauges for rainfall quantification in Gothenburg, Sweden. *Earth System Science Data*, *14*(12), 5411-5426.

Results - June 2nd 2015

$$r_{avg} \xrightarrow{K\to\infty} \left(\frac{A_r^{max} + \mathcal{O}(1)}{a_{cal}^{max} \cdot L}\right)^{\frac{1}{b}} \approx \left(\frac{A_r^{max}}{a_{cal}^{max} \cdot L}\right)^{\frac{1}{b}}$$

$$a_{cal}^{max} = a \cdot (\ln(K) + \gamma)^b$$



RG Cumulative: 20.0 mm

CML Cumulative: 22.9 mm

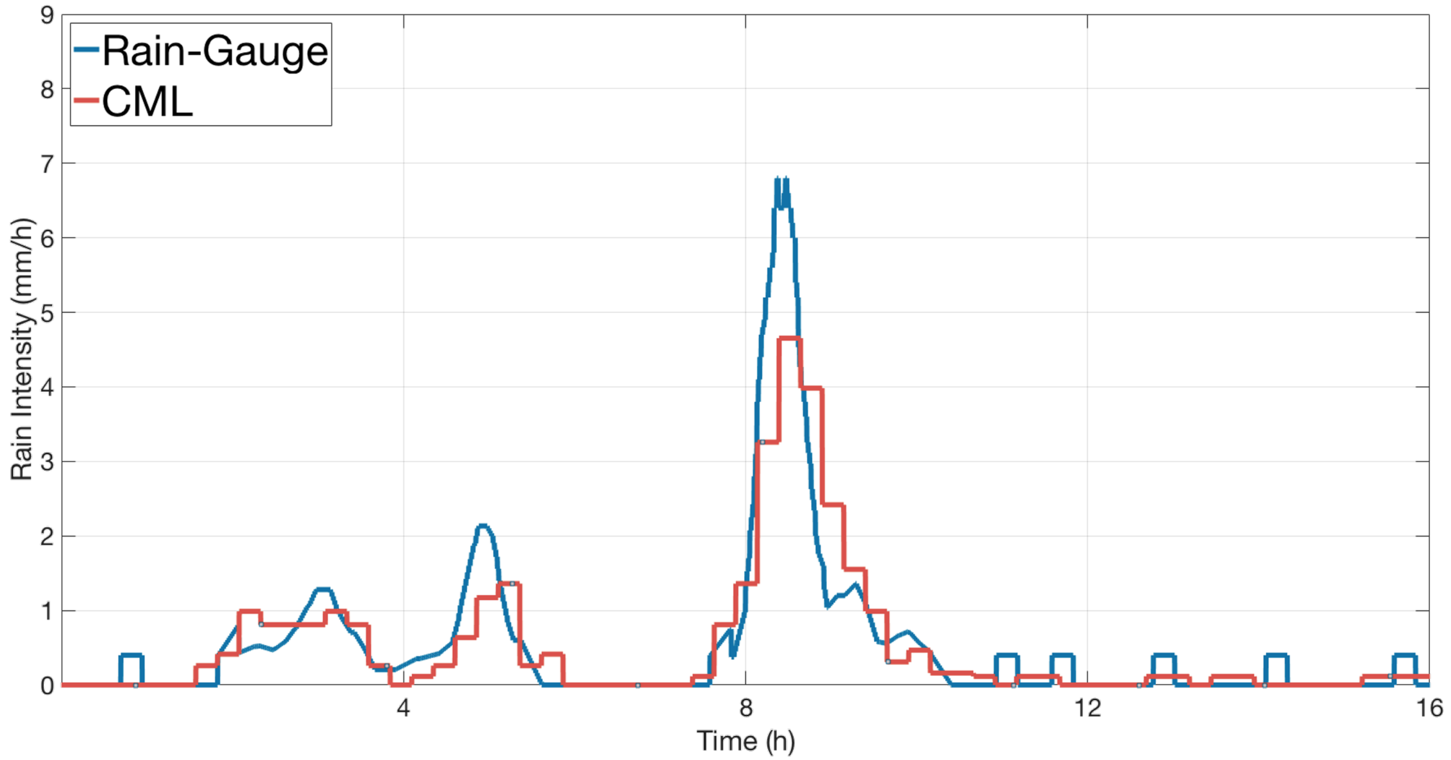
Corr: 0.89

Ostrometzky, J. (2017). *Statistical signal processing of extreme attenuation measurements taken by commercial microwave links for rain monitoring*. Tel Aviv, Israel: Tel Aviv University.

Results – July 25th 2015

$$r_{avg} \xrightarrow{K\to\infty} \left(\frac{A_r^{max} + \mathcal{O}(1)}{a_{cal}^{max} \cdot L}\right)^{\frac{1}{b}} \approx \left(\frac{A_r^{max}}{a_{cal}^{max} \cdot L}\right)^{\frac{1}{b}}$$

$$a_{cal}^{max} = a \cdot (\ln(K) + \gamma)^b$$



RG Cumulative: 8.7 mm

CML Cumulative: 8.1 mm

Corr: 0.93

Ostrometzky, J. (2017). *Statistical signal processing of extreme attenuation measurements taken by commercial microwave links for rain monitoring*. Tel Aviv, Israel: Tel Aviv University.

Full Results

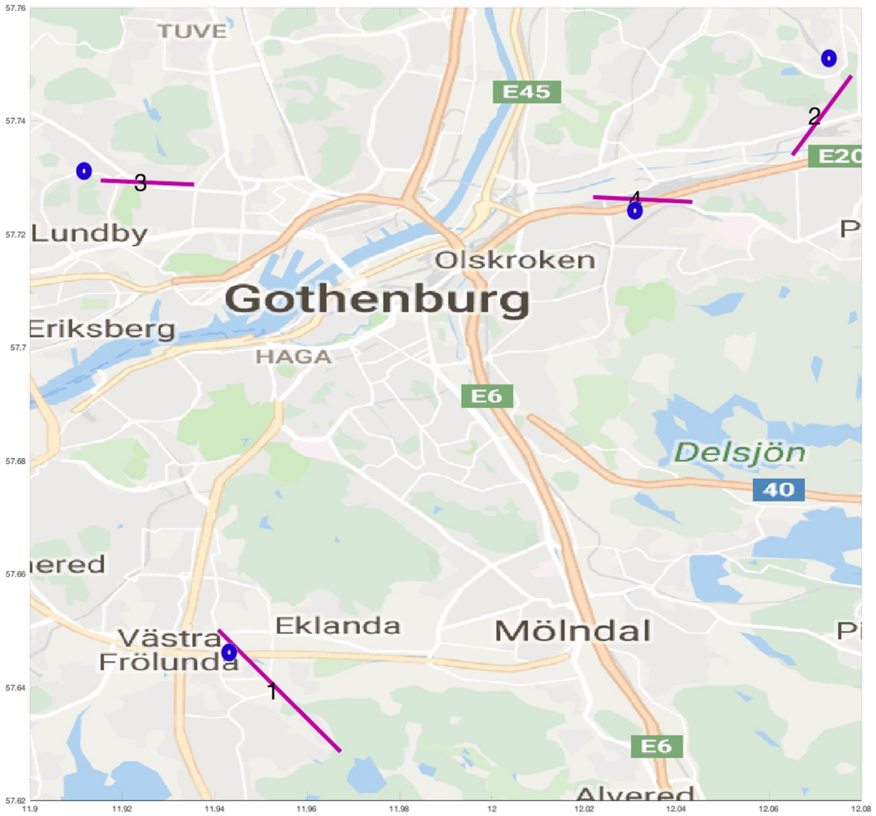
Four (double) CMLs
and four nearby rain-gauges

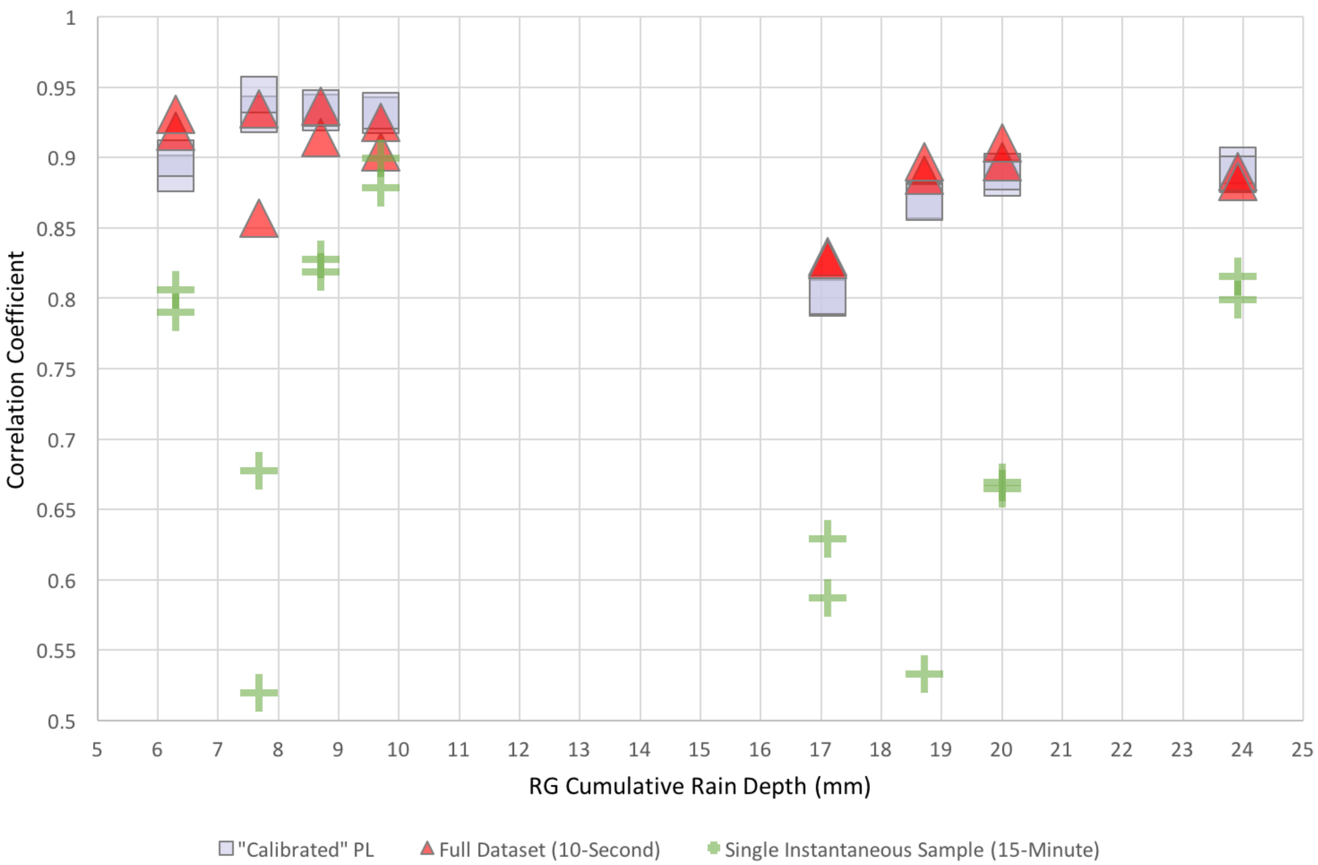
Event 1: June 2nd 2015
Event 2: July 25th 2015

Overall :
16 different rain estimates

Andersson, J. C., Olsson, J., Van de Beek, R., & Hansryd, J. (2022). OpenMRG: Open data from Microwave links, Radar, and Gauges for rainfall quantification in Gothenburg, Sweden. *Earth System Science Data*, *14*(12), 5411-5426.

Full Results – Correlation





Ostrometzky, J. (2017). *Statistical signal processing of extreme attenuation measurements taken by commercial microwave links for rain monitoring*. Tel Aviv, Israel: Tel Aviv University.

Take Away Message

Min-Max samples at 15-minute intervals, although "just" two samples, hold significantly more information than two instantaneous samples, and can potentially outperform instantaneous samples at higher sampling rate (e.g., once every 10 minutes, once every 5 minutes, etc.), for rain monitoring.

References

- Ostrometzky, J., & Messer, H. (2020). On the information in extreme measurements for parameter estimation. *Journal of the Franklin Institute*, *357*(1), 748-771.
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- Ostrometzky, J., & Messer, H. (2017). Dynamic determination of the baseline level in microwave links for rain monitoring from minimum attenuation values. *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, *11*(1), 24-33.
- Ostrometzky, J. (2017). *Statistical signal processing of extreme attenuation measurements taken by commercial microwave links for rain monitoring*. Tel Aviv, Israel: Tel Aviv University.
- Habi, Hai Victor, and Hagit Messer. "Recurrent neural network for rain estimation using commercial microwave links." *IEEE Transactions on Geoscience and Remote Sensing* 59.5 (2020): 3672-3681.
- Andersson, J. C., Olsson, J., Van de Beek, R., & Hansryd, J. (2022). OpenMRG: Open data from Microwave links, Radar, and Gauges for rainfall quantification in Gothenburg, Sweden. *Earth System Science Data*, *14*(12), 5411-5426.

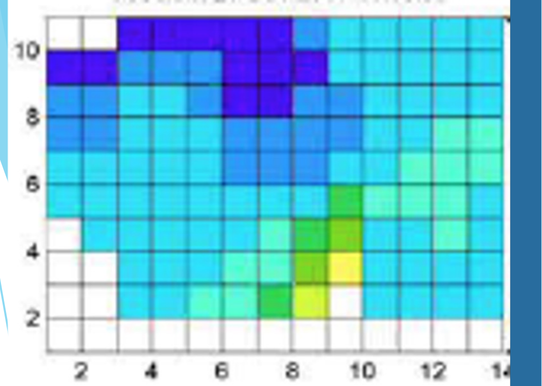
3. How is the CMLs network topology related to the rain-field reconstruction resolution?

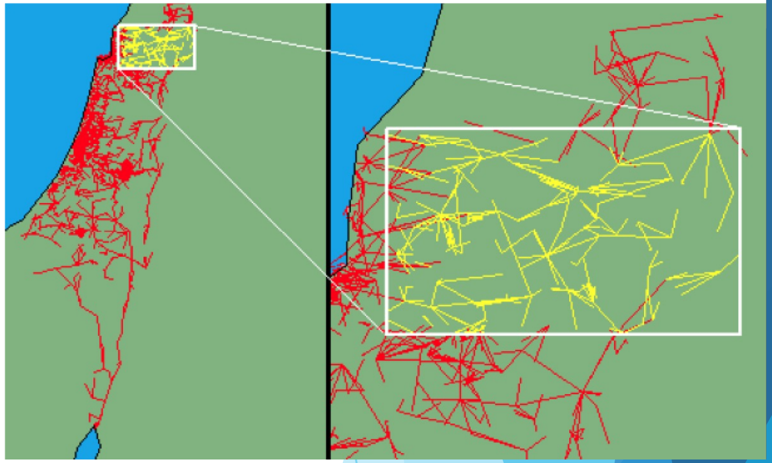
The Resolution Question

- Given a set of n CMLs, what is the achievable resolution of 2-D rainfall mapping?
- The answer depends on:

1. The rain-field characteristics

2. The number and the topology of the links



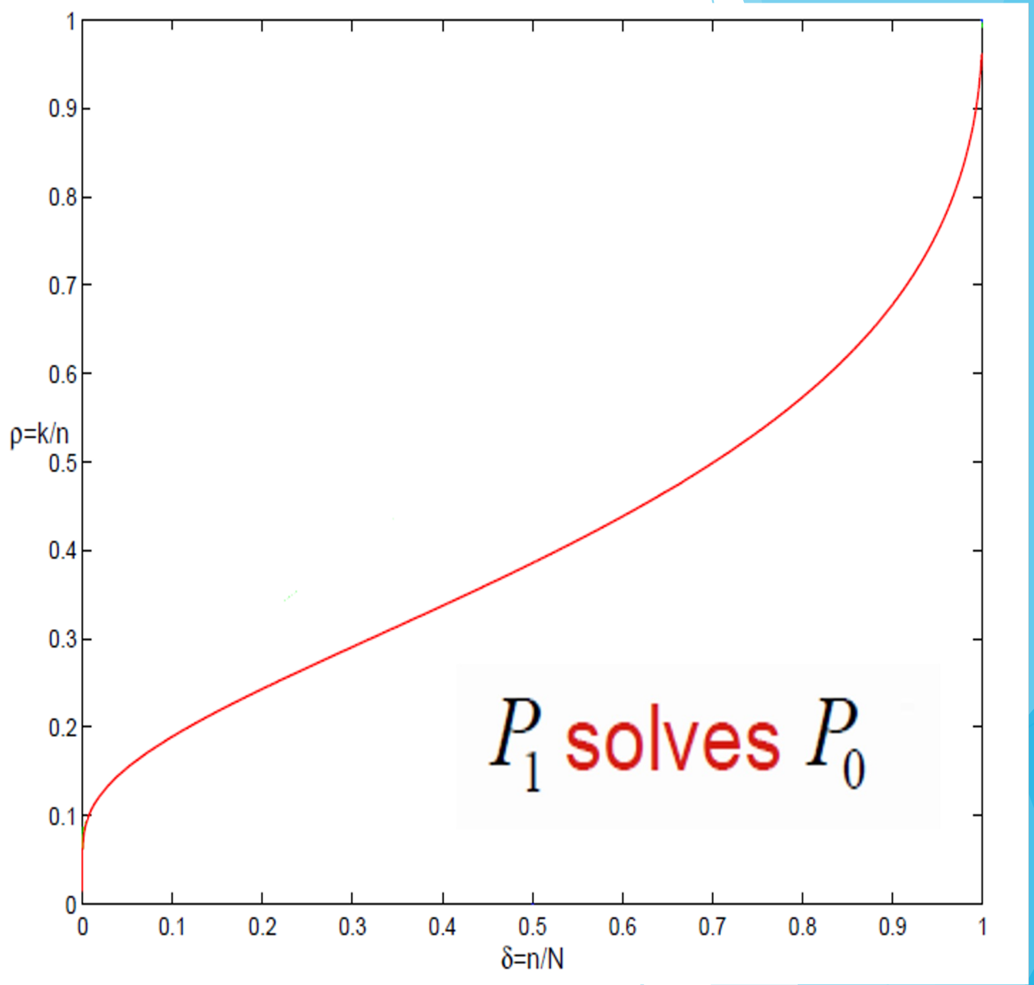


- We propose a stochastic approach, where the rain-field is characterized by its sparsity, and the CMLs are characterized by their spatial distribution.

Gazit, Lior, and Hagit Messer. "Sufficient Conditions for Reconstructing 2-D Rainfall Maps." *IEEE Transactions on Geoscience and Remote Sensing* 99 (2018): 1-10.

The phase-transition diagram (Donoho-Tanner)

- Solving $\underline{y}$=A$\underline{x}$ where A is an nXN matrix and n<N
- If A is a *random* matrix (e.g., the elements are standard Gaussian) and if $\underline{x}$ is sparse (k out of n entries are non-zero at random locations), then a phase transition exists.
- That is, under the curve, $\underline{x}$ can be retrieve with high probability



The ***compressed sensing*** *solution*:

(P_n) min_x||x||n s.t. y=Ax

The Method

- In our (approximated) problem $\underline{y}$=A$\underline{x}$, A is indeed random but:

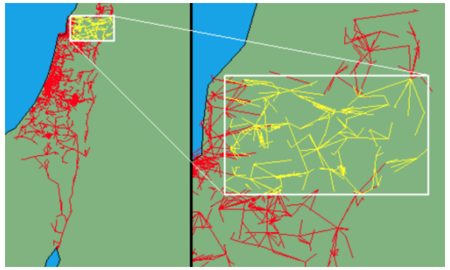
(a) pixels (from the same link) are dependent;

(b) The matrix distribution is not Gaussian

- We have:

1. Studied the statistics of link's topology: Gazit, Lior, and Hagit Messer. "Advancements in the Statistical Study, Modeling, and Simulation of Microwave-Links in Cellular Backhaul Networks." *Environments* 5.7 (2018): 75.
2. Found an appropriate sparsifying transformation (Haar)
3. Studied empirically the existence of a phase transition

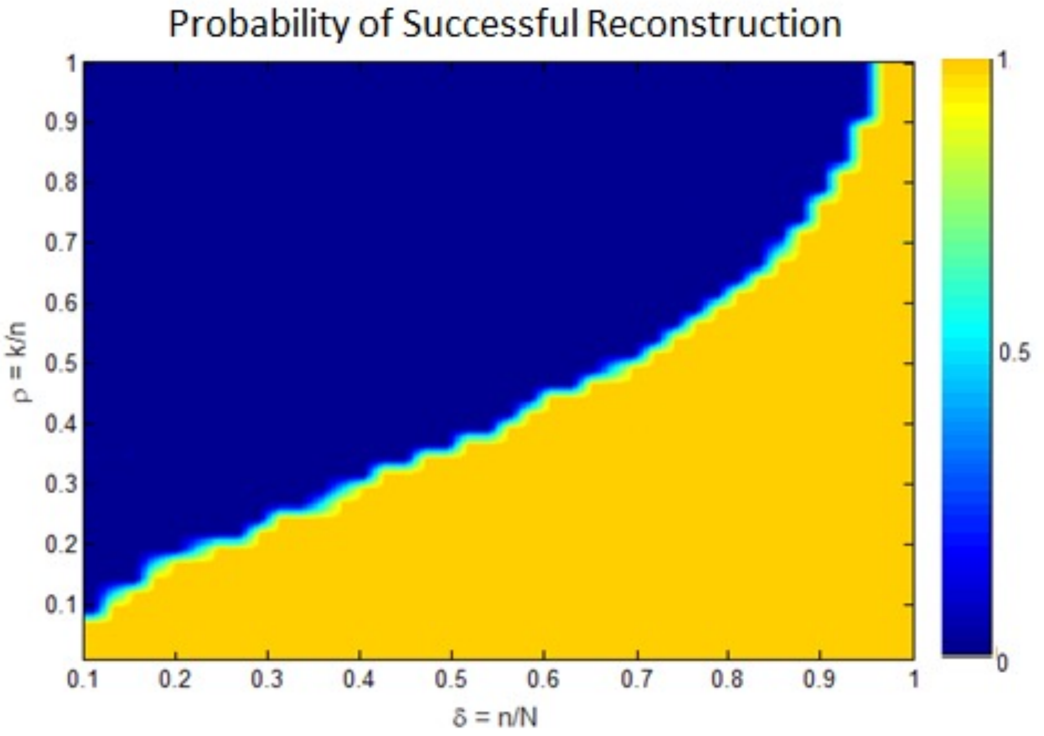
Empirical Results



δ – *degree of underdetermination, inversly proportional to resolution*
ρ – *degree of sparsity*

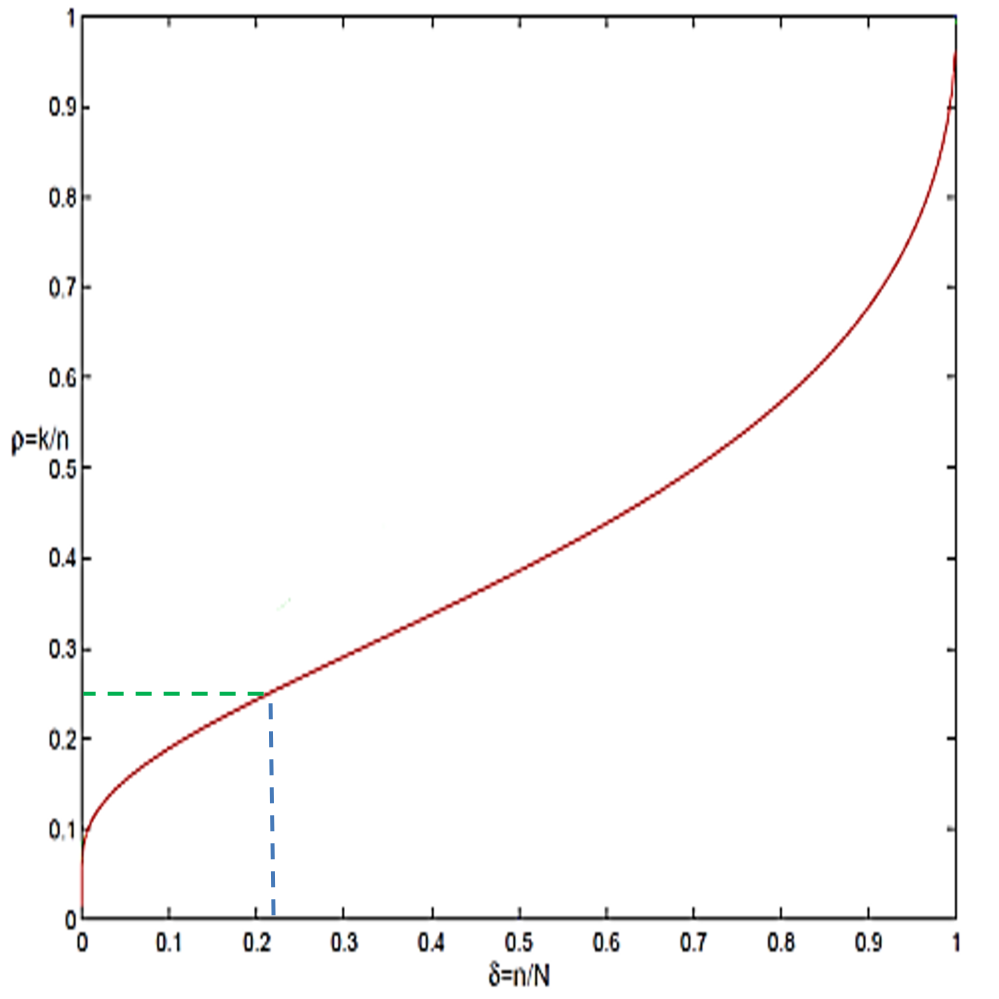
This empirical test is valid under the assumptions that the links are uniformly distributed in the area of interest, where the orientation of each link is also uniformly distribution, and its length is exponentially distributed

N is proportional to the resolution, so given the sparsity and the number of links, it can be set to guarantee constructability



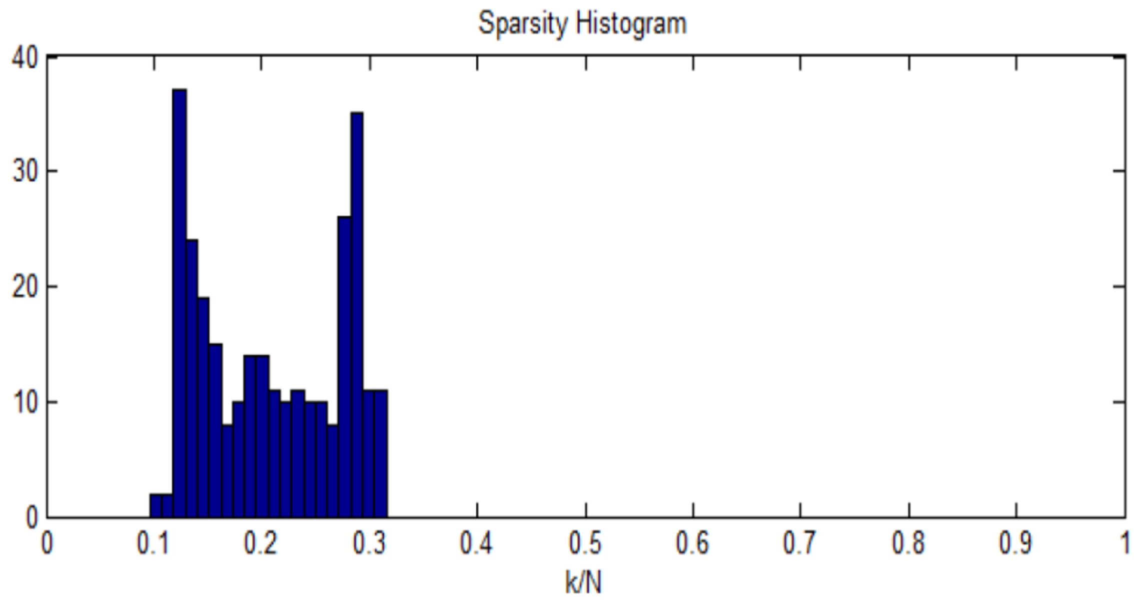
Example of use

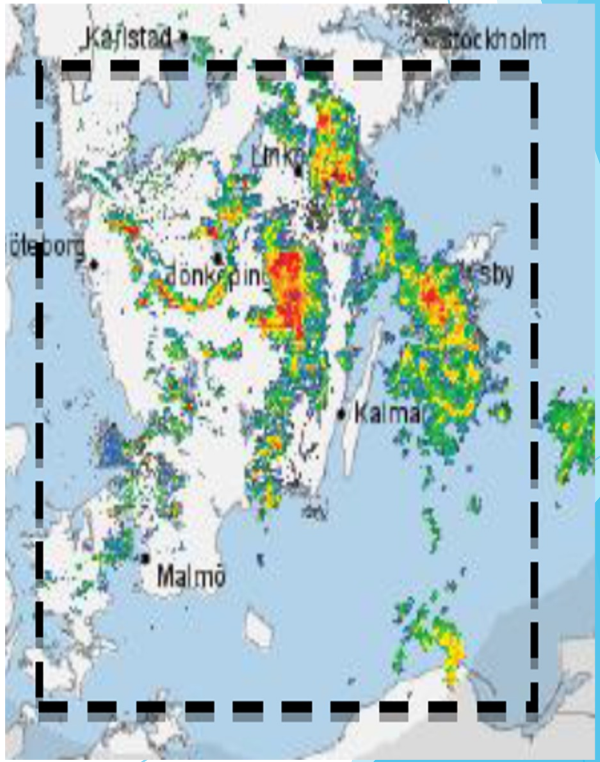
- Assume that you have n=200 hops (links), uniformly distributed at an area of 15X15 km.
- Suppose that the desired resolution is 0.5X0.5km, meaning that N=900 and n/N=0.22
- Then, reconstruction of rain fields with sparsity up to ρ =0.25, (that is, 0.25X0.22=5.5% non-zero pixels – directly, or after Haar Transform), is ensured.



Meaning?

Study the sparsity of 288 radar images in Goteborg (Sweden) shows that a typical values of k/N are between 0.12 and 0.3.





Conclusions

- Signal processing is a powerful tool which enables theoretical study of inherent limitations and algorithms' design considerations in practical applications under statistical modeling.
- We had a sneak peek at a few practical examples in our field of opportunistic sensing of rain.
- But there are many, many more!☺

Thank You!

https://cellenmonlab.tau.ac.il/

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